Weighted Haar Wavelet-Like Basis for Scattering Problems

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***Abstract*—A class of wavelet-like basis functions orthonormal to the oscillatory functions with spatial frequency near the free space propagation constant is introduced to solve the scattering of a transition matrix (TM)-polarized plane wave due to a metallic strip. The electric field integral equation (EFIE) for the unknown surface current distribution is formulated. The method of moments with rectangular pulse basis functions and point matching is applied to discretize the integral equation into a matrix equation. The dense impedance matrix is transformed to a sparse matrix using compact support wavelet-like basis functions. The effects of the discretization size on the performance of the wavelet-like basis functions are presented.**

I. Introduction

In the last few years, wavelet analysis has drawn a great attention in both applied mathematics and many engineering disciplines because of its multiresolution property [1]. When wavelets are employed as a basis set, the solution of integral equation arising in electromagnetics can be sped up by changing the dense impedance matrices to sparse matrices [2], [3]. Problems still arise, however, because most of the wavelets developed by mathematics community [4], [5] are not tailored for electromagnetic problems, which involve oscillatory kernels. For example, in order to reduce the oscillation of the discretized impedance matrix, the discretization of 0.03λ, which is much smaller than the conventional discretization size, 0.1λ, was used in [6] to solve a 90° dihedral corner reflector under a transition matrix (TM)-polarized plane-wave incident.

In this letter, a class of wavelet-like basis functions is introduced to solve the scattering of a TM-polarized plane wave due to a metallic strip. The basis functions have the following properties: 1) compact support with different length scale and 2) all but k basis orthonormal to k weighting functions.

II. Theory

A metallic strip illuminated by a TM-polarized plane wave is shown in Fig. 1. The z-directed surface current distribution $J_z(\vec{r}')$ on the strip is related to the incident field by an electric field integral equation (EFIE)

$$\frac{\omega\mu}{4}\int_L J_z(\vec{r}')H_0^{(2)}(k_0|\vec{r}-\vec{r}'|)dr' = E_z^i \quad r \in L \quad (1)$$

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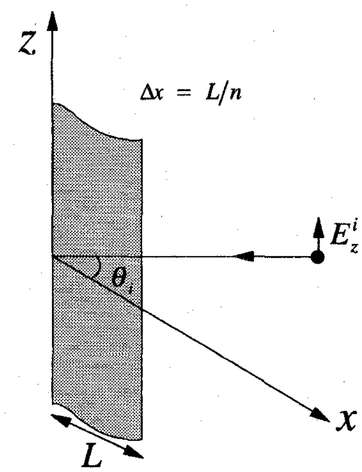

Fig. 1. A TM-polarized plane wave incident on a metallic strip.

where L represents the surface of the strip. Expanding the surface current distribution $J_z(\vec{r}')$ in uniform rectangular pulse basis and point matching at the centers of the pulse basis x_i, $i = 1, 2, \cdots, n$, the resulting equation gives

$$[Z][a] = [V] \quad (2)$$

where $[a]$ is the column matrix for the coefficient of the current basis and $[Z]$ is the dense impedance matrix.

The matrix equation (2) is then transformed into wavelet domain $[Z'][a'] = [V']$ where $[Z'] = [U][Z][U]^T, [a'] = [U][a]$, and $[V'] = [U][V]$ and the rows of $[U]$ are wavelet-like basis. Because of the compact support property of the wavelet-like basis, there are only $kn[1 \log_2(n/k) + 1]$ nonzero elements in the matrix U where k is the number of weighting functions used so that the number of operations for the multiplication of the dense impedance matrix with the wavelet basis matrix could be implemented by either a sparse matrix solver or a tailored routine.

In this letter, the wavelet-like basis is constructed based on the multiresolution decomposition of the weighted Haar functions. For a discrete set of points $\{x_1, x_2, \cdots, x_n\}$ where $n = 2^m k$, k and m are positive integers. A k-dimensional vector space V_0 spanned by the Haar functions weighted with functions $W_j(x_i)$ is defined as

$$V_0 = \text{span}\{\langle W_j(x_1), W_j(x_2), \cdots, W_j(x_n)\rangle \mid j = 1, 2, \cdots, k\}. \quad (3)$$

We define another $2k$-dimensional vector space V_{-1}, which is also spanned by the weighted Haar functions $W_j(x_i)$, $j =$

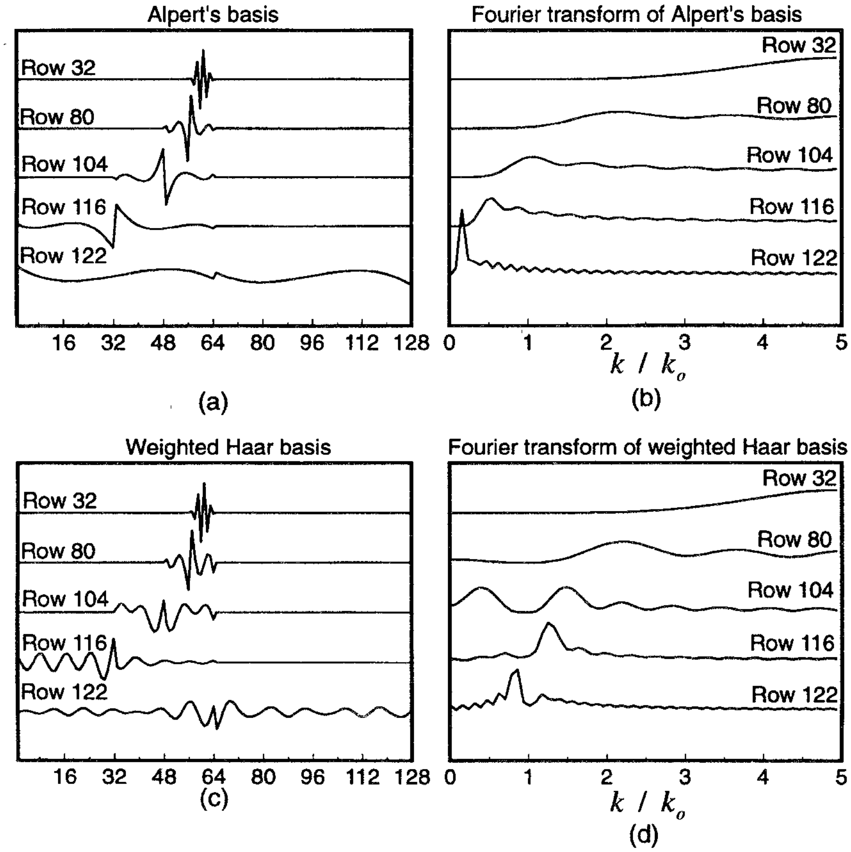

Fig. 2. Wavelet-like basis with different support, $n = 128$ discretization points, and $k = 4$ and their Fourier transform. (a) Alpert's wavelets, (b) weighted Haar wavelets, (c) Fourier transform of the Alpert's wavelets, and (d) Fourier transform of the weighted Haar wavelets.

$1, 2, \cdots, k$, on $\{x_1, x_2, \cdots, x_{n/2}\}$ and $\{x_{n/2+1}, x_2, \cdots, x_n\}$. The decomposition process is repeated until we get the vector space V_{-m}, which is the entire n-dimensional vector space. As the vector space V_{-i} is a subspace of vector space V_{-i-1}, i.e.,

$$V_0 \subset V_{-1} \subset \cdots \subset V_{-m} \tag{4}$$

an orthogonal compliment T_{-i} of V_{-i} is defined such that $V_{-i-1} = T_{-i} \oplus V_{-i}$. Therefore, the entire vector space V_{-m} can be decomposed as follows:

$$\begin{aligned} V_{-m} &= T_{1-m} \oplus V_{1-m} \\ &= T_{1-m} \oplus T_{2-m} \oplus V_{2-m} \\ &= T_{1-m} \oplus T_{2-m} \oplus T_{3-m} \oplus V_{3-m} \\ &\bullet \\ &\bullet \\ &= T_{1-m} \oplus T_{2-m} \oplus T_{3-m} \oplus \cdots \oplus T_{-1} \oplus V_{-1} \\ &= T_{1-m} \oplus T_{2-m} \oplus T_{3-m} \oplus \cdots \oplus T_{-1} \oplus T_0 \oplus V_0. \end{aligned} \tag{5}$$

The implementation of the multiresolution decomposition is based on the Gram–Schmidt orthogonalization process.

III. Results

In this section, the proposed wavelet-like basis are employed to transform the impedance matrix for the scattering problem. The weighted functions $W_j(x_i)$ used are $\cos[k_0 L \cdot (1 - 0.4\frac{(j-3)}{2k})\frac{x_i}{x_n}]$, $j = 1, 2, \cdots, 2k$. Some weighted wavelet basis vectors using the oscillatory functions as weighting functions are shown in Fig. 2(c)–(d). The Alpert's wavelet basis vectors, in which the polynomial x_i^{j-1} are used as the weighting function, are also shown in Fig. 2(a) and (b). It shows that some basis vectors of Alpert's wavelet have significant components in high spatial frequency, so that the amplitude of the impedance matrix elements projected on these vector are large. For the weighted wavelet-like basis, the spatial frequency components near k_0 are reduced.

The transformed impedance matrix is thresholded by zeroing elements of the impedance matrix with magnitude less than a tolerance times the maximum element. The percentage of sparsity (S) and percentage relative error (ε) are defined by

$$S = \frac{n_0}{n^2} \times 100\% \tag{6}$$

and

$$\varepsilon = \frac{\|J_z - J_z'\|}{\|J_z\|} \times 100\% \tag{7}$$

where n_0 is the zero elements after thresholding, J_z' is the surface current distribution obtained with thresholding, and $\|\cdot\|$ denotes the L^2 norm. A typical transformed matrix obtained using the weighted Haar wavelet-like basis is shown in Fig. 3. In Fig. 4, the sparsity versus solution error of the problem is plotted for both weighted wavelet-like basis functions and Alpert's wavelet-like basis functions. Its shows that the sparsity of the matrix using Alpert's wavelet-like basis is less than 20% when the discretization size is 0.2λ. The results show

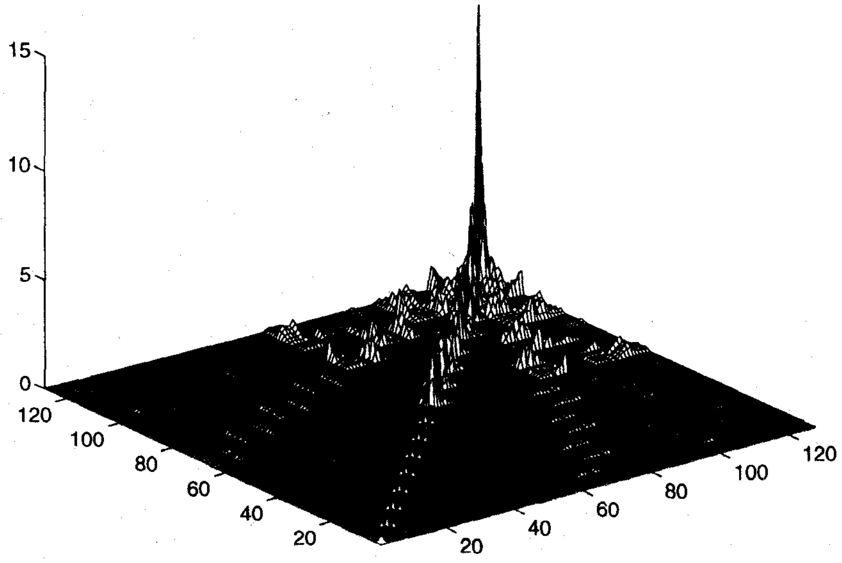

Fig. 3. Wavelet-domain impedance matrix for a TM-polarized plane wave incident on a metallic strip with length $12.8\lambda_0$. The strip is discretized into 128 pulses and $k = 8$ is used.

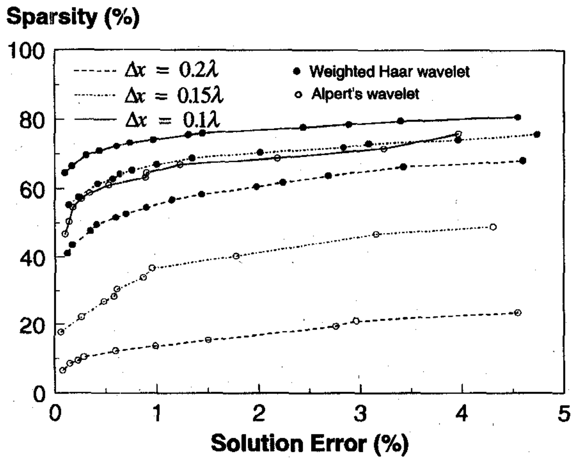

Fig. 4. Sparsity of the wavelet domain impedance matrix for a TM-polarized plane wave incident on a metallic strip as a function of solution error. $n = 128, k = 8, \Delta x = 0.1, 0.15$ and 0.2λ. Solid circle: weighted Haar wavelet; hollow circle: Alpert's wavelet.

that the nonzero elements could be increased using tailored wavelet basis instead of using the conventional wavelet basis with vanishing moments, such as Alpert's wavelet basis and Daubechies' wavelet basis.

IV. Conclusion

The use of the weighted Haar wavelet-like basis to reduce the impedance matrix for a TM-polarized plane wave incident on a metallic strip is demonstrated. The sparsity of the present problem using weighted Haar wavelet-like basis is higher than using Alpert's wavelet basis especially for large discretization size. In addition, the use of tailored wavelet basis should be extended to two- or even three-dimensional problems.

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